AD-A082 322 OHIO STATE UNIV COLUMBUS DEPT OF GEODETIC SCIENCE F/G 8/5
POTENTIAL COEFFICIENT AND ANOMALY DEGREE VARIANCE MODELLING REV--ETC(U)
SEP 79 R H RAPP F19628-79-C-0027
UNCLASSIFIED DGS-293 AFGL-TR-79-0245 NL

1 OF 1
AD
A082322

END
DATE
FILMED
4-80
DTIC

(14) DGS-293, SCIENTIFIC-2

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(19) REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: (18) AFGL TR-79-0245
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): (6) POTENTIAL COEFFICIENT AND ANOMALY DEGREE VARIANCE MODELLING REVISITED
5. TYPE OF REPORT & PERIOD COVERED: Scientific Report No. 2
6. PERFORMING ORG. REPORT NUMBER: Dept. of Geod. Sci. No. 293
7. AUTHOR(s): (10) Richard H. Rapp
8. CONTRACT OR GRANT NUMBER(s): (15) F19628-79-C-0027
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Geodetic Science, The Ohio State University - 1958 Neil Avenue, Columbus, Ohio 43210
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F, (16) 2309G1AW (17) G1
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Geophysics Laboratory, Hanscom AFB, Massachusetts 01731, Contract Monitor: Bela Szabo
12. REPORT DATE: (11) September 1979
13. NUMBER OF PAGES: ~~14~~ (12) 23
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS (of this report): Unclassified

15a. DECLASSIFICATION DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report)

A - Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Geodesy, gravity, collocation, covariances.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The decay of the power spectrum of the earth's gravitational potential is modeled using the GEM 10B solution of NASA and a solution complete to degree 180 at OSU. The modelling was first done using the form of A/ℓ^B where A and B were parameters determined by various adjustment techniques. For degrees 3 through 36, the best estimate of A is 10.6×10^{-6} and B = 2.17 based on the GEM 10B data. Earlier models by Kaula and Tscherning/Rapp contain too much power as compared to the actual field beyond degree 8. If B were fixed at either 2 or 2.5, the best fit for the

DD FORM 1473, 1 JAN 73 — EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

next page

405 22

cont. range 3 to 36 is obtained with B = 2.

Anomaly degree variance models were also computed from the anomaly degree variances of the 180 solution and other data such as a point anomaly variance, and a horizontal gradient variance. With this data, we found that the parameters of the earlier Tscherning/Rapp model are still valid provided one is willing to accept a high gradient variance. Excellent fit to all data types is obtained with the two component model suggested by Moritz and described in detail by Jekeli. All models used in the study, and models by Wagner/Colombo and Heller/Jordan are compared in terms of common parameters such as point anomaly variance, gradient variance, undulation variance and fits to observed potential coefficient variations.

Foreword

This report was prepared by Richard H. Rapp, Professor, Department of Geodetic Science, The Ohio State University, under Air Force Contract No. F19628-79-C-0027, The Ohio State University Research Foundation Project No. 711664. The contract covering this research is administered by the Air Force Geophysics Laboratory, Hanscom Air Force Base, Massachusetts, with Mr. Bela Szabo, Contract Monitor.

Computer programs written by L. Krieg and C. Jekeli were very useful in the computations described in this report.

Table of Contents

Introduction

The earth's gravitational potential can be expressed in a spherical harmonic series where the potential coefficients are $\overline{C}_{\ell m}$, $\overline{S}_{\ell m}$ where the overbar indicates full normalization (Kaula, 1966, Heiskanen and Moritz, 1967). The potential degree variances are computed from:

$$\sigma_\ell^2 = \sum_{m=0}^{\ell} (\overline{C}_{\ell m}^2 + \overline{S}_{\ell m}^2) \tag{1}$$

Estimates of the variation of σ_ℓ were obtained from gravimetry by Kaula (1966) who suggested the following behavior:

$$\sigma_\ell = \frac{10^{-5}}{\ell^2} \tag{2}$$

Another term of interest is the anomaly degree variance:

$$c_\ell = \gamma^2(\ell-1)^2 \sum_{m=0}^{\ell} (\overline{C}_{\ell m}^2 + \overline{S}_{\ell m}^2) \tag{3}$$

where γ is an average value of gravity over the earth.

Estimates of σ_ℓ and c_ℓ are needed for several purposes. We can gain information on the density structure of the earth by considering various mass distributions that would imply a potential coefficient variation of the form as equation (2) (Kaula, 1977). We can also compute covariance functions of quantities related to the earth's gravity field if we have models of the anomaly degree variances.

The need for some simple formulas to describe σ_ℓ or c_ℓ have led to a number of studies that have fitted empirically determined data to various models. Such procedures were followed by Rapp (1973) who used limited satellite implied information and limited terrestrial gravity information. A more refined model was described in Tscherning and Rapp (1974) that became widely used as the basis for covariance computations needed for the prediction of quantities dependent on the gravity field using least squares collocation. Moritz (1977) suggested an improved model for the anomaly degree variances that was investigated by Jekeli (1978) and Hein and Jochemczyk (1979). Wagner and Colombo (1978) analyzed altimeter data to imply some simple rules for σ_ℓ that depended on the range of ℓ being investigated. Heller and Jordan (1979) used the superposition of white noise spherical shells at various depths with various masses to generate a disturbing potential.

Recently, several questions have arisen that can be answered with new data that is now available. Specifically, we are interested to see if the real world potential coefficients do fall off as $1/\ell^2$, or could it be $1/\ell^{2.5}$, or some other power. Are the parameters of the c_ℓ model described by Tscherning and Rapp (1974) substantially valid today? Is the model suggested by Moritz (1977) a better fit to existing data? Do the suggestions of Wagner and Colombo (1978) and Heller and Jordan (1979) fit our new data?

The data that we will consider are the potential coefficients to degree 36 from the GEM 10B field (Lerch et. al., 1978), and the potential coefficients to degree 180 based on an adjusted set of 64800 $1^\circ \times 1^\circ$ mean free air anomalies described by Rapp (1978).

Potential Coefficient Modelling

The first modelling will be done with the following equation:

$$\sigma(C_{\ell m}, S_{\ell m}) = \frac{A \times 10^{-6}}{\ell^{B}} \tag{4}$$

so that for Kaula's rule $A = 10$ and $B = 2$. Shown in Figure 1 are the values of σ from GEM 10B, from Kaula's rule of thumb and as implied by the c_ℓ model of Tscherning and Rapp. These two models have clearly more power in them than in the actual gravitational field.

We then carried out least squares fits to (4) solving for A and B that best fit the values from GEM 10B and the 180,180 solution in two different minimization procedures. The first procedure minimized the sum of the squares of the residuals while the second procedure effectively minimized the sum of the squares of the percentage residuals. (A percent residual would be 100 (adjusted value - observed value)/(observed value).) This latter procedure is more effective when fitting to data having wide variations in magnitudes. We give in Table 1 values of A and B for different weighting cases. We see that the use of weights as $1/\text{obs}^2$ yields poorer residual fits but slightly better percentage fits. The differences between the GEM 10B and the 180,180 solutions do not appear significant. Plots of GEM 10B and the two GEM 10B fits are shown in Figure 2.

Table 1. Model Parameters for Equation (4) by Fitting Observed Potential Coefficient Variation in the Range of Degree 3 to 36.

Solution	A	B	RMS Residual x 10^6	RMS % Residual	Weighting
GEM 10B	10.6	2.17	0.026	16.3	$1/\text{obs}^2$
GEM 10B	13.2	2.26	0.014	16.8	equal
180,180	9.41	2.12	0.037	16.5	$1/\text{obs}^2$
180,180	13.8	2.30	0.014	19.9	equal

We have also carried out fittings to equation (4) where we have fixed B to be 2 or 2.5 exactly and solved for A for the two ranges of degree 3 to 20 and 3 to 36. The results are given in Table 2.

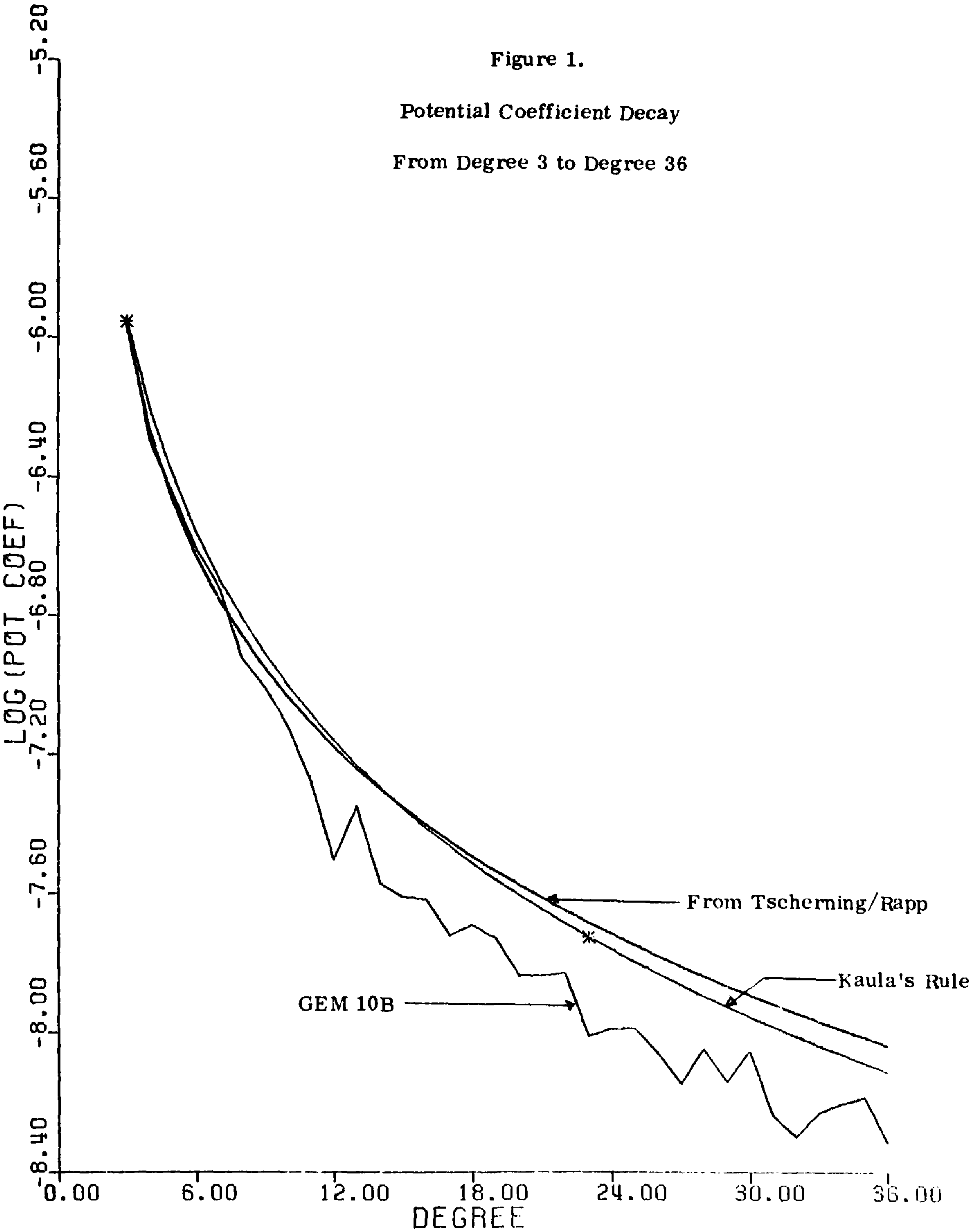


Figure 1.

Potential Coefficient Decay

From Degree 3 to Degree 36

Figure 2.

Model Fits to GEM 10B

$$\sigma(C_\ell, S_\ell) = \frac{A \times 10^{-6}}{\ell^B}$$

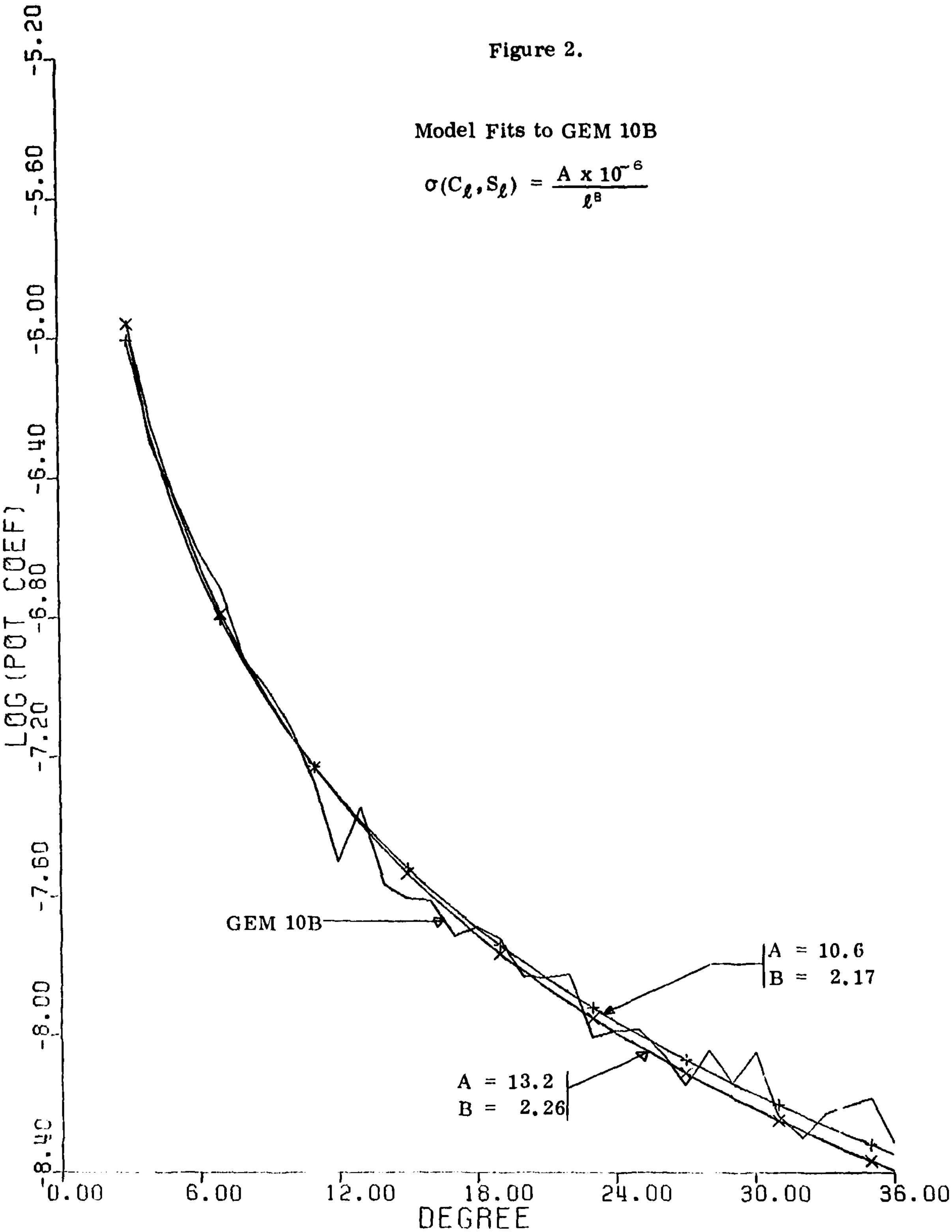


Table 2. Fitting to Equation (4) Enforcing Parameter B Using the GEM 10B Potential Coefficients

Range	A	B*	RMS Residual x 10^{6}	RMS % Residual	Weighting
3-20	7.02	2.0	0.086	22.4	1/obs^2
3-20	9.57	2.0	0.032	50.0	equal
3-20	22.3	2.5	0.087	18.6	1/obs^2
3-20	17.8	2.5	0.025	24.0	equal
3-36	6.68	2.0	0.070	19.0	1/obs^2
3-36	9.57	2.0	0.024	53.3	equal
3-36	27.0	2.5	0.012	28.1	1/obs^2
3-36	17.8	2.5	0.019	36.6	equal

* Fixed Value

If we consider the degree range 3 to 20, the best percentage fit is obtained when B is 2.5. If we consider the range 3 to 36, the best percentage fit is obtained when B is 2.0. Thus, the answer to the question as to how the potential coefficients decay vis 2.0 or 2.5 seems to depend on the degree range being considered. However, even in the range degree 3-20 the use of $B = 2.5$ is only slightly better than the 2.0 value. To graphically display the differences, we show in Figure 3, the case 3 to 36 with $B = 2$ and $B = 2.5$ with weighting as 1/obs^2. It is clear that at the higher degrees, the solution with $B = 2$ gives a better fit to the data than the $B = 2.5$ case.

We next consider the potential coefficient variation implied by the solution to degree 180. We show in Figure 4 this variation along with the variation implied by Kaula's rule of thumb and the Tscherning/Rapp anomaly degree variance model. At low degrees, the actual field has less power than the two models. The Tscherning/Rapp model has more power at all degrees, and the Kaula rule has less power after degree 70. In Figure 5, we show again the 180,180 variation along with the Wagner and Colombo (1978) model and the five shell model of Heller and Jordan (1979). The Wagner and Colombo model consists (for the range of ℓ considered here) of five different functions of the form of equation (4). The Wagner and Colombo model fit our data much better than the data of Heller and Jordan.

We have fitted equation (4) to the data with results shown in Table 3 for the case of two parameters being found, and in the case B is fixed at 2 or 2.5.

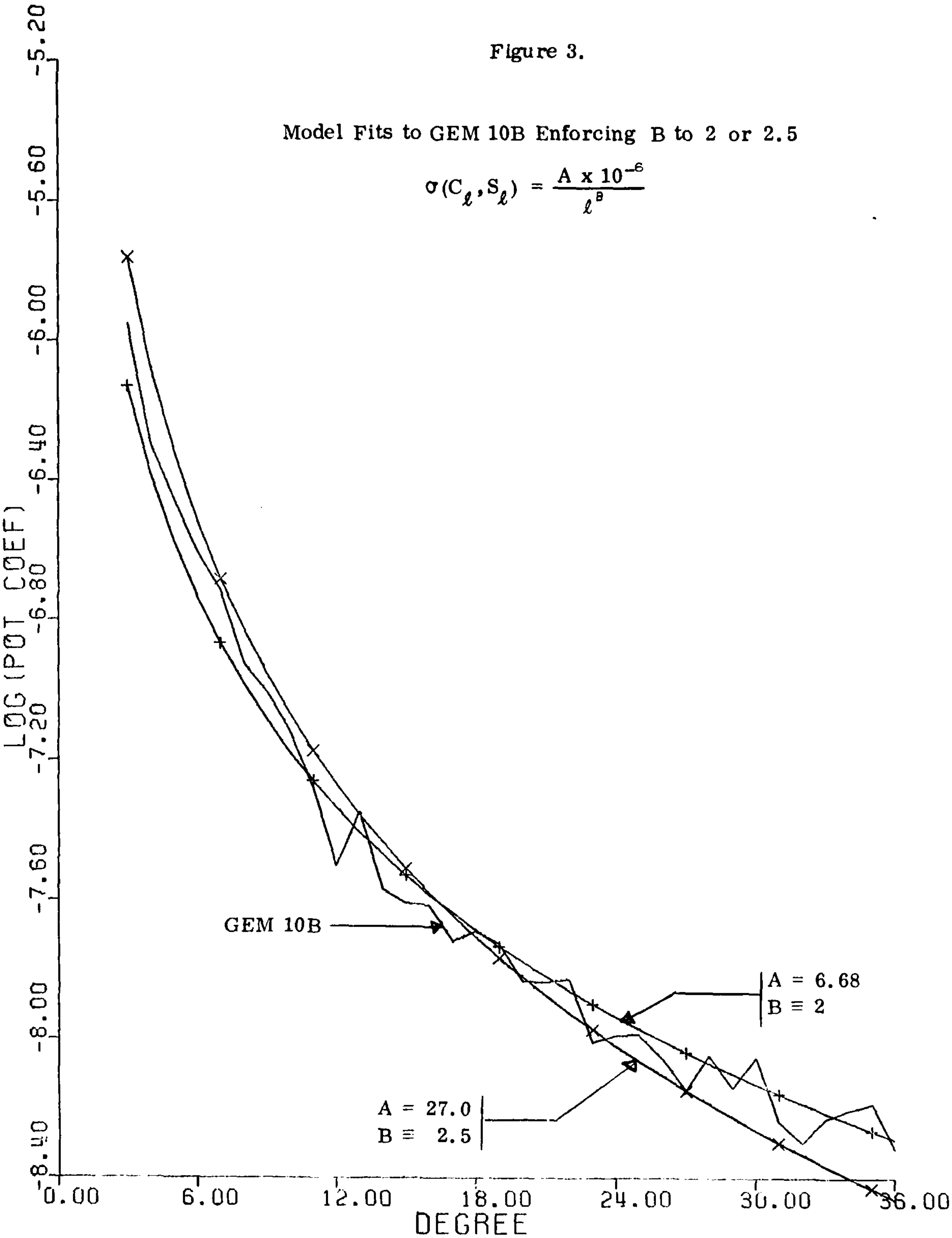


Figure 3.
Model Fits to GEM 10B Enforcing B to 2 or 2.5
σ(C_ℓ, S_ℓ) = A x 10^-6 / ℓ^B
GEM 10B
A = 6.68
B ≡ 2
A = 27.0
B ≡ 2.5
LOG(POT COEF)
-5.20
-5.60
-6.00
-6.40
-6.80
-7.20
-7.60
-8.00
-8.40
0.00
6.00
12.00
18.00
24.00
30.00
36.00
DEGREE

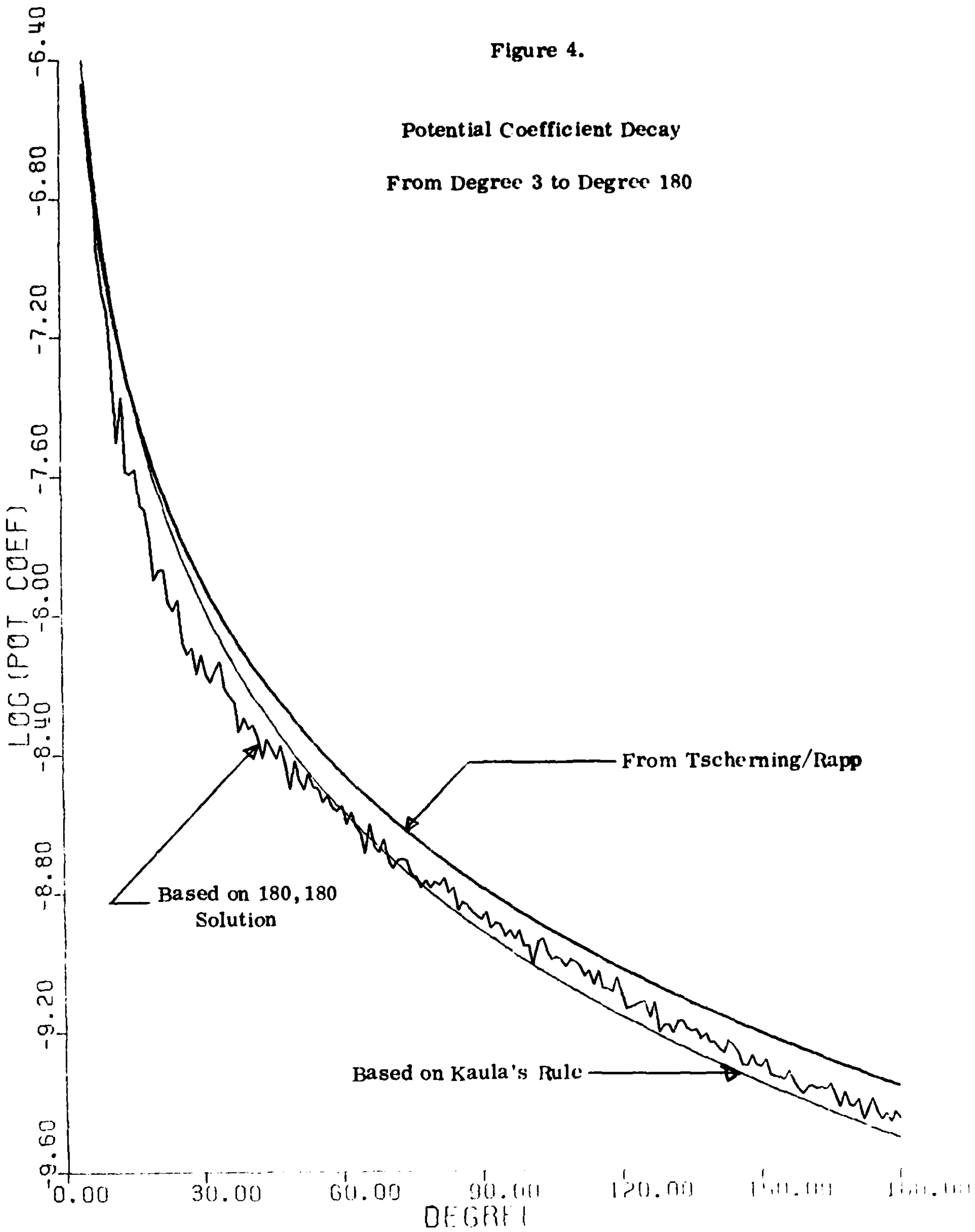


Figure 4.

Potential Coefficient Decay

From Degree 3 to Degree 180

Figure 5.

Potential Coefficient Decay

From Degree 3 to Degree 180

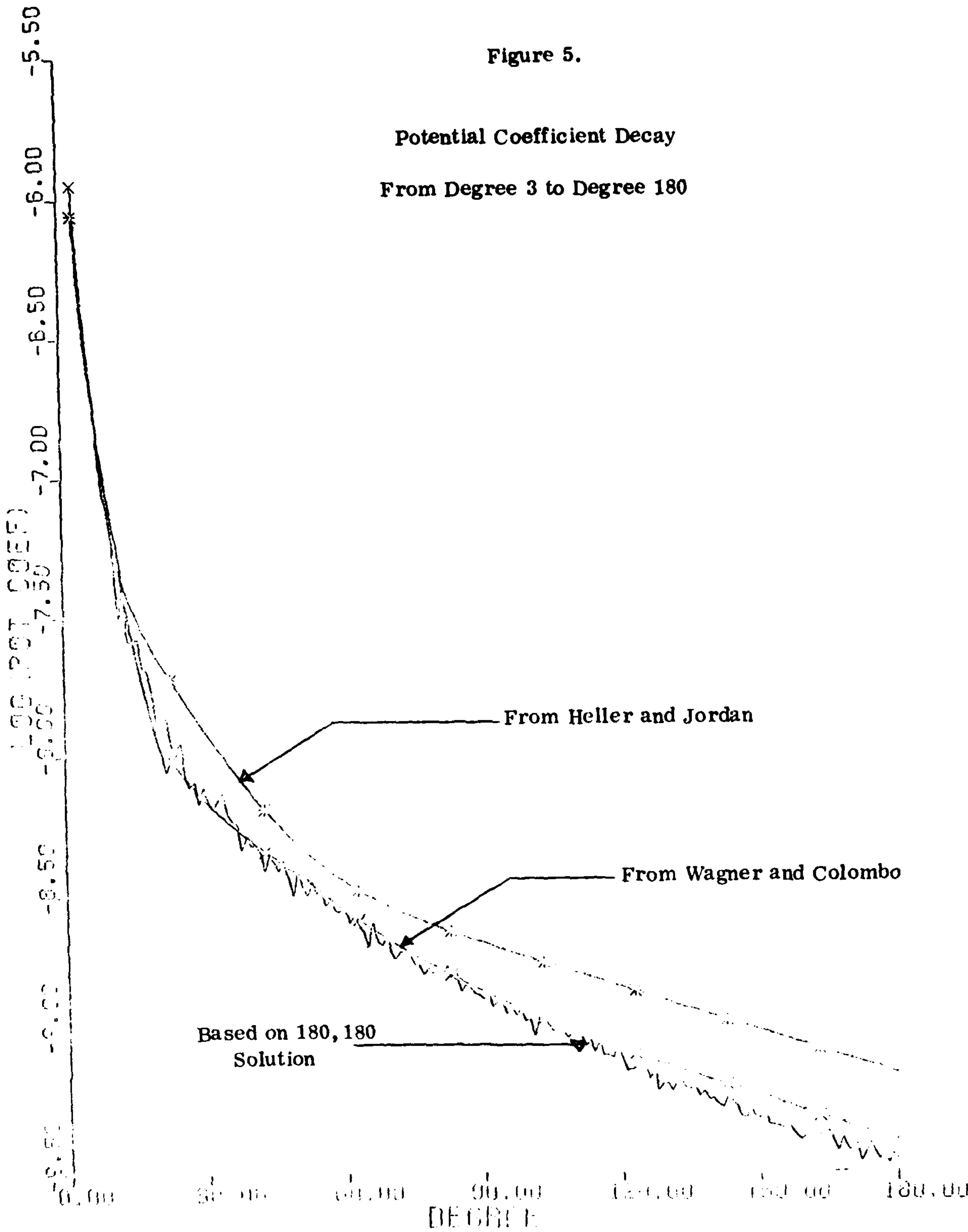


Table 3. Model Parameters for Equation (4) from Fitting Observed Potential Coefficient Variations in the Range of Degree 3 to 180

A	B	RMS Residual x 10^6	RMS % Residual	Weighting
4.10	1.80	0.045	13.8	1/obs^2
13.7	2.29	0.006	59.8	equal
9.71	2.0*	0.011	26.6	1/obs^2
9.54	2.0*	0.011	26.0	equal
81.6	2.5*	0.036	106	1/obs^2
17.7	2.5*	0.007	76.0	equal

* Value fixed in the adjustment.

We show in Figure 6 plots of the fits for the first two solutions where we see that the solution with the minimum percent residual fits best at the higher degrees. In the case of B fixed at 2 there is no substantial difference in the results from either weighting scheme. When B is taken as 2.5, the fits are substantially poorer than when B is 2.

Anomaly Degree Variance Modelling

Anomaly degree variances can be computed from potential coefficients using equation (3). (More rigorous equations are discussed in Jekeli (1978).) The Tscherning/Rapp model took the form:

$$c_\ell = \frac{A\,(\ell - 1)}{(\ell - 2)\,(\ell + B)} \tag{5}$$

where B is a positive integer. In addition to A and B as parameters, an additional term, s, was introduced such that the sum of the c_ℓ times $s^{\ell+2}$ would yield the point anomaly variance, C_0, at the surface of the earth. We have:

$$C_0 = \sum_{\ell}^{\infty} c_\ell \, s^{\ell+2} \tag{6}$$

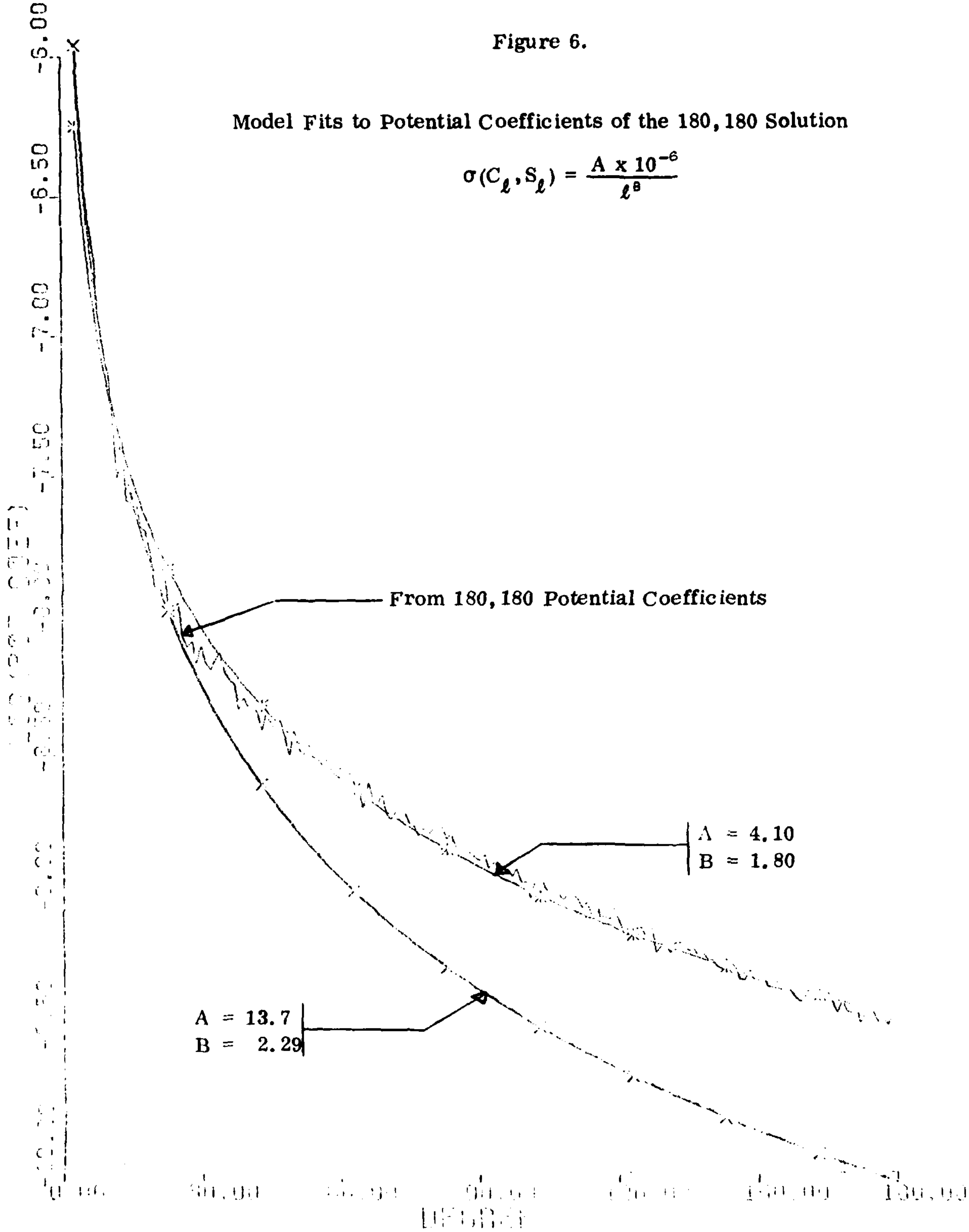
In Tscherning/Rapp, the parameters were $A = 425.28$ mgal^2, $B = 24$, and $s = 0.999617$. These quantities were obtained by fitting the model (from degree 3) to observed anomaly degree variances to degree 20, 1° and 5° block anomaly variances, and a point anomaly variance of 1795 mgal^2 with respect to an ellipsoidal reference field.

For this paper, we have extended this fitting to the anomaly degree variances to degree 180 but have excluded the block variances because they have become extraneous information. We have introduced information on the horizontal gradient

Figure 6.

Model Fits to Potential Coefficients of the 180, 180 Solution

$$\sigma(C_\ell, S_\ell) = \frac{A \times 10^{-6}}{\ell^B}$$



variance G_{OH} for help in strengthening the solution. The horizontal gradient variance is related to the anomaly degree variances by (Jekeli, 1978):

$$G_{OH} = \frac{1}{2}\sum_{\ell=2}^{\infty}\frac{(\ell+2)^2}{R^2}\, c_\ell \; s^{\ell+3} \tag{7}$$

(Pertinent discussions on the interpretation of R, c_n, and s may be found in Jekeli (ibid).) However, we have adopted for use with equation (5) only, a $G_{OH} = 3500\ E^2$ which is that implied by the Tscherning/Rapp model. This value is too high in terms of the real world as pointed out by Moritz (1977) but as shown by Jekeli (1978), the Tscherning/Rapp model is not compatible with a low horizontal gradient variance.

We therefore made a parameter fit to the observed anomaly degree variances from degree 3 to 180, a point anomaly variance of 1800 ± 25 mgal^2 and a horizontal gradient variance of $3500 \pm 50\ E^2$. The computer program used was originally written by Jekeli for his earlier studies. A number of different runs were made with different B values and weighting schemes. Two solutions of specific interest are detailed in Table 4. Besides giving C_0, G_{OH}, L_0 (the undulation variance) of the model, we give, in the table, the correlation length which is defined as ξ where $C(\xi) = C_0/2$.

Table 4. Parameters, Residuals, and Adjusted Values Obtained in Fitting to the Anomaly Degree Variance Model of Tscherning/Rapp

	Solution One	Solution Two
A	135.19 mgal^2	429.48 mgal^2
B	5	24
s	0.999780	0.999613
RMS c_ℓ Residual	1.35 mgal^2	3.08 mgal^2
RMS % Residual	49	108
C_0*	838 mgal^2	1801 mgal^2
G_{OH}	3424 E^2	3500 E^2
L_0†	599 m^2	617 m^2
ξ	65 km	43 km

* Add 7.6 mgal^2 to refer to ellipsoidal model.
† Add 314 m^2 to refer to ellipsoidal model.

Solution One was made to give the best overall fit to the anomaly degree variances. Solution Two was made to fit the anomaly degree variances as well as possible and to yield a point anomaly variance of about 1800 mgal^2. This latter solution has a B value the same as the original Tscherning/Rapp model, with the A and s values being only slightly different. Because of this agreement, we consider that the parameters used in the earlier model are still valid provided we are willing to accept the high gradient variance.

These anomaly degree variance models imply a potential coefficient variation found by substituting (1) into (3). Plots of these variations are shown in Figure 7. Solution Two shows systematically more power than the observed field while Solution One shows less power after about degree 15.

In order to avoid the high gradient variance associated with the c_ℓ model given as equation (5) Moritz (1977) suggested a two component model. Jekeli (1978) slightly recast this model into the following form:

$$c_\ell = \alpha_1 \frac{\ell - 1}{\ell + A} \sigma_1^{\ell+2} + \alpha_2 \frac{\ell - 1}{(\ell - 2)(\ell + B)} \sigma_2^{\ell+2} \tag{8}$$

where α_1, α_2, σ_1, σ_2, A and B are quantities to be determined. In practice, A and B are integers that are varied while the other quantities are solved for in an adjustment procedure. This procedure was implemented by Jekeli (1978) who found parameters based on available data which included the c_ℓ values implied by the GEM 9 potential coefficients, a C_0, and G_{0H} values. The c_ℓ values given in (8) will refer to a sphere whose radius is the mean radius of the earth. Consequently, the point anomaly variance on the surface of this sphere can be found by substituting (8) into (6) and setting s to 1.

The fitting to the c_ℓ model given in equation (8) was carried out using the given anomaly degree variances to degree 180, a $C_0 = 1800$ mgals^2, and a $G_{0H} = 800$ E^2. Initial fits with G_{0H} as used by Jekeli (1978) and Moritz (1977) did not yield good data fits. The value of 800 E^2 was chosen considering the observations described by Hein et. al., (1979). However, G_{0H} still remains a weak link in our model development.

Approximately 25 different runs were made with different A and B values and different weighting schemes. (In these fits the anomaly degree variances were computed from (3) without any further reduction to a mean sphere.) We give the results of these fits in Table 5 for two solutions. Case One is the solution that gives the best overall fit to the data, and Case Two is the one that best fits the observed anomaly degree variances. These two solutions, in terms of potential coefficient variations, are shown in Figure 8. At high degrees, we see that Case One has more power than the actual field or Case Two. This occurs in this model to satisfy the need for a C_0 of about 1800 mgal^2.

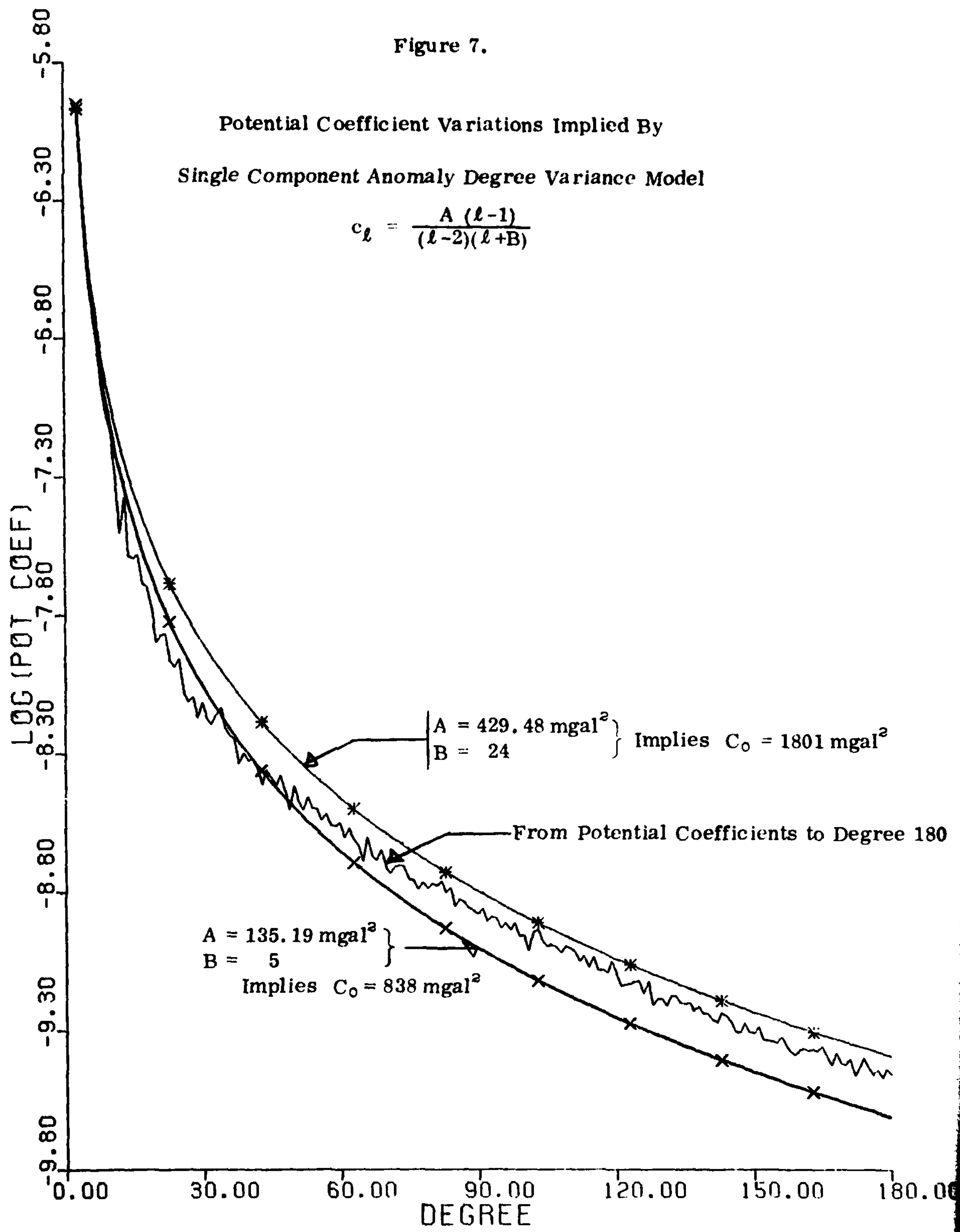


Figure 7.
Potential Coefficient Variations Implied By
Single Component Anomaly Degree Variance Model
$c_\ell = \frac{A\,(\ell-1)}{(\ell-2)(\ell+B)}$
A = 429.48 mgal²
B = 24
Implies C_0 = 1801 mgal²
From Potential Coefficients to Degree 180
A = 135.19 mgal²
B = 5
Implies C_0 = 838 mgal²
LOG (POT COEF)
-5.80
-6.30
-6.80
-7.30
-7.80
-8.30
-8.80
-9.30
-9.80
0.00
30.00
60.00
90.00
120.00
150.00
180.00
DEGREE

Figure 8.

Potential Coefficient Variations Implied By

The Two Component Anomaly Degree Variance Model of Moritz

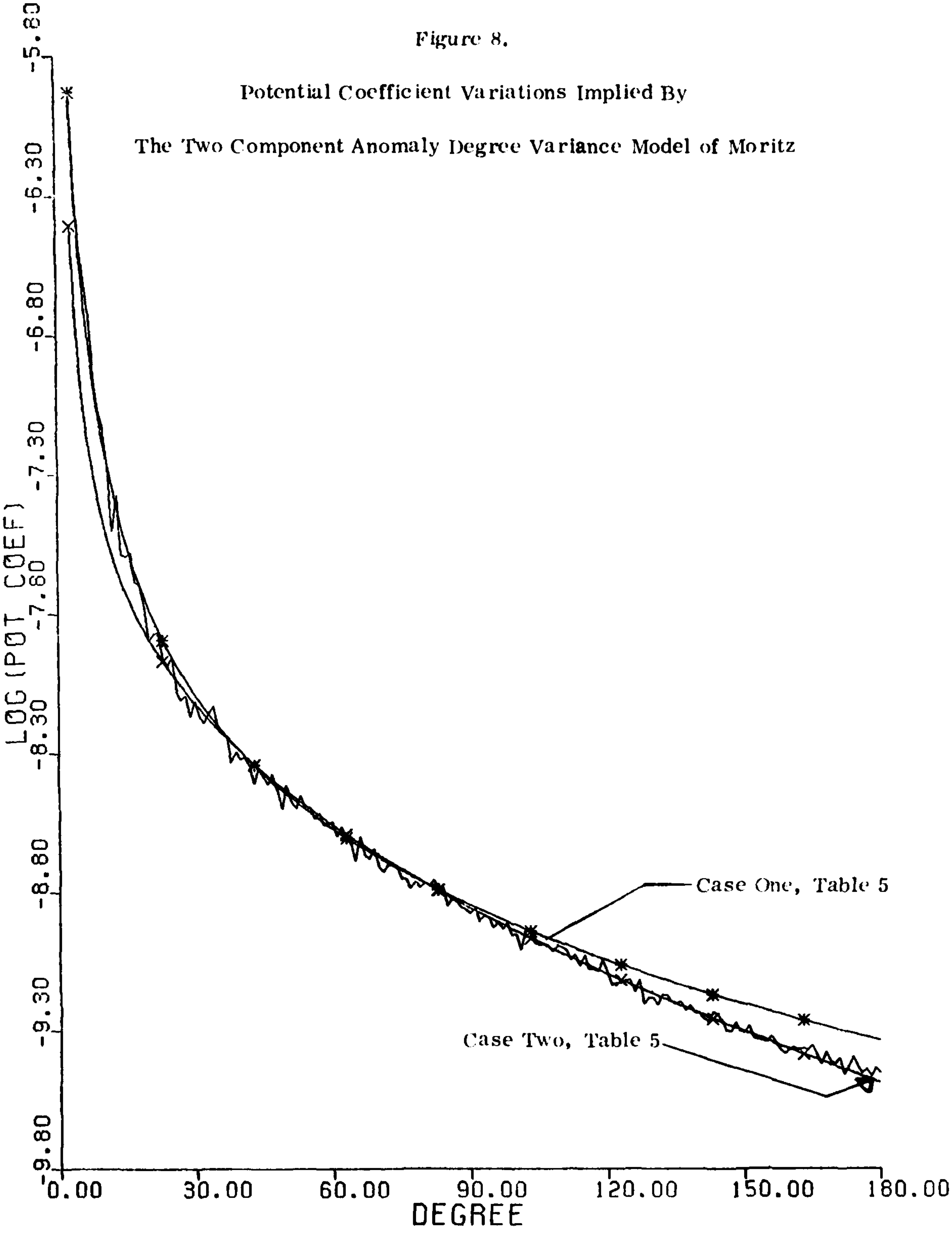


Table 5. Parameters, Residuals, and Adjusted Values Obtained in Fitting to the Anomaly Degree Variance Model of Moritz

	Case One	Case Two
α_1	3.4050 $mgal^2$	14.966 $mgal^2$
α_2	140.03 $mgal^2$	999.25 $mgal^2$
σ_1	.998006	.987969
σ_2	.914232	.850000++
A	1	75
B	2	20
RMS c_ℓ Residual	1.08 $mgal^2$	0.93 $mgal^2$
RMS % Residual	39.2	15.1
C_0*	1794 $mgal^2$	647 $mgal^2$
G_{OH}	1053 E^2	15 E^2
L_0+	616 m^2	641 m^2
ξ	24 km	110 km

* Add 7.6 $mgal^2$ to refer to ellipsoidal model.
\+ Add 314 m^2 to refer to ellipsoidal model.
++ Constrained to this value.

Summary and Conclusions

In this paper, we have tried to improve our knowledge of potential coefficient and anomaly degree variance behavior by comparing new data to existing models and by obtaining new parameters of various models. The primary new information going into these models is a set of potential coefficients given to degree 180 based on satellite derived potential coefficients, terrestrial gravity anomalies, and anomalies derived from Geos-3 altimeter data.

One of the first steps was to compare the potential coefficient variations of the GEM 10B solution to the variations implied by the Kaula rule of thumb and the Tscherning/Rapp degree variance model. We found that both models have significantly more power than exists in the real gravity field. Two parameter fits to the data were carried out. Other fits were carried out to see if the potential coefficients decay as $1/\ell^2$ or $1/\ell^{5/2}$. Choosing the criteria as a best percentage fit, the best fit is obtained with a fall off of $1/\ell^2$ for ℓ between 3 and 36. For ℓ between 3 and 20, a fall off of $1/\ell^{5/2}$ is slightly better than $1/\ell^2$.

We then made comparisons and fits to a number of different potential coefficient and anomaly degree variance models from ℓ equal 3 to 180. We summarize these results in Table 6 for a number of common quantities.

Table 6. Summary of Model Comparisons and Fits With and To the Potential Coefficient Solution to Degree 180

Model	Number of Parameters	Pot. Coeff. Residuals RMS x 10^6	Pot. Coeff. Residuals %	Variances* C_0 ($mgal^2$)	Variances* G_{OH} E^2	Variances* L_0 (m^2)	Correlation Length (km)
Kaula	2	.0126	28	$1111^{\dagger}$	$1600^{\dagger}$	$705^{\dagger}$	
Tscherning/Rapp	3	.0074	38	1787	3500	612	42
Heller/Jordan	10	.0200	75	1816	255	556	66
Wagner/Colombo	12	.0078	14	$1193^{\dagger}$	$1656^{\dagger}$	$642^{\dagger}$	
A/ℓ^B, Case 1**	2	.0449	14	$1556^{\dagger}$	$8210^{\dagger}$	$216^{\dagger}$	
A/ℓ^B, Case 2**	2	.0061	60	$271^{\dagger}$	$28^{\dagger}$	$594^{\dagger}$	
One comp. c_ℓ model, Case $1^{\dagger\dagger}$	3	.0074	39	1801	3500	617	43
One comp. c_ℓ model, Case $2^{\dagger\dagger}$	3	.0048	25	838	3424	599	65
Two comp. c_ℓ model, Case $1^{*\dagger}$	6	.0072	17	1794	1053	616	24
Two comp. c_ℓ model, Case $2^{*\dagger}$	6	.0640	16	647	15	641	110

* From Degree 3.

** Case 1: $A = 4.10 \times 10^{-6}$, $B = 1.80$; Case 2: $A = 13.7 \times 10^{-6}$, $B = 2.29$.

†† Case 1: $A = 429.48$ $mgal^2$, $B = 24$; Case 2: $A = 135.19$ $mgal^2$, $B = 5$.

*† See Table 4.

† $s = 0.999617$

Of the existing models, the Wagner/Colombo model seems best although the anomaly variance is less than the 1800 $mgal^2$ we feel is reasonable. Neither case of the four A/ℓ^B fits all quantities well although the fit to potential coefficient variations is quite good for case 1. The fits of the one component degree variance model do not appear significantly better than the original Tscherning/Rapp model. The two component model, case 1, seems to give the best overall fit to the data we have knowledge of, although the correlation length seems to be shorter than one usually expects. However, it agrees quite well with the estimate of 22 km obtained from the anomaly covariance function given in Table 19 of Rapp (1964).

We have also computed a 1° covariance function from the two component degree variance model ($A = 1$, $B = 2$) which is shown in Figure 9. We also show the empirically determined 1° covariance function from Tscherning and Rapp (1974, Table 15) with both functions scaled to have the same variance. The agreement is quite good, especially at the location of the zero crossings.

We specifically conclude that for covariance studies not dealing with accuracy estimations in gradient computations, the parameters of the Tscherning/Rapp model are still valid. For a better fit to all expected values, the fit to the two component model suggested by Moritz (specifically case 1 in Table 6) seems to be best.

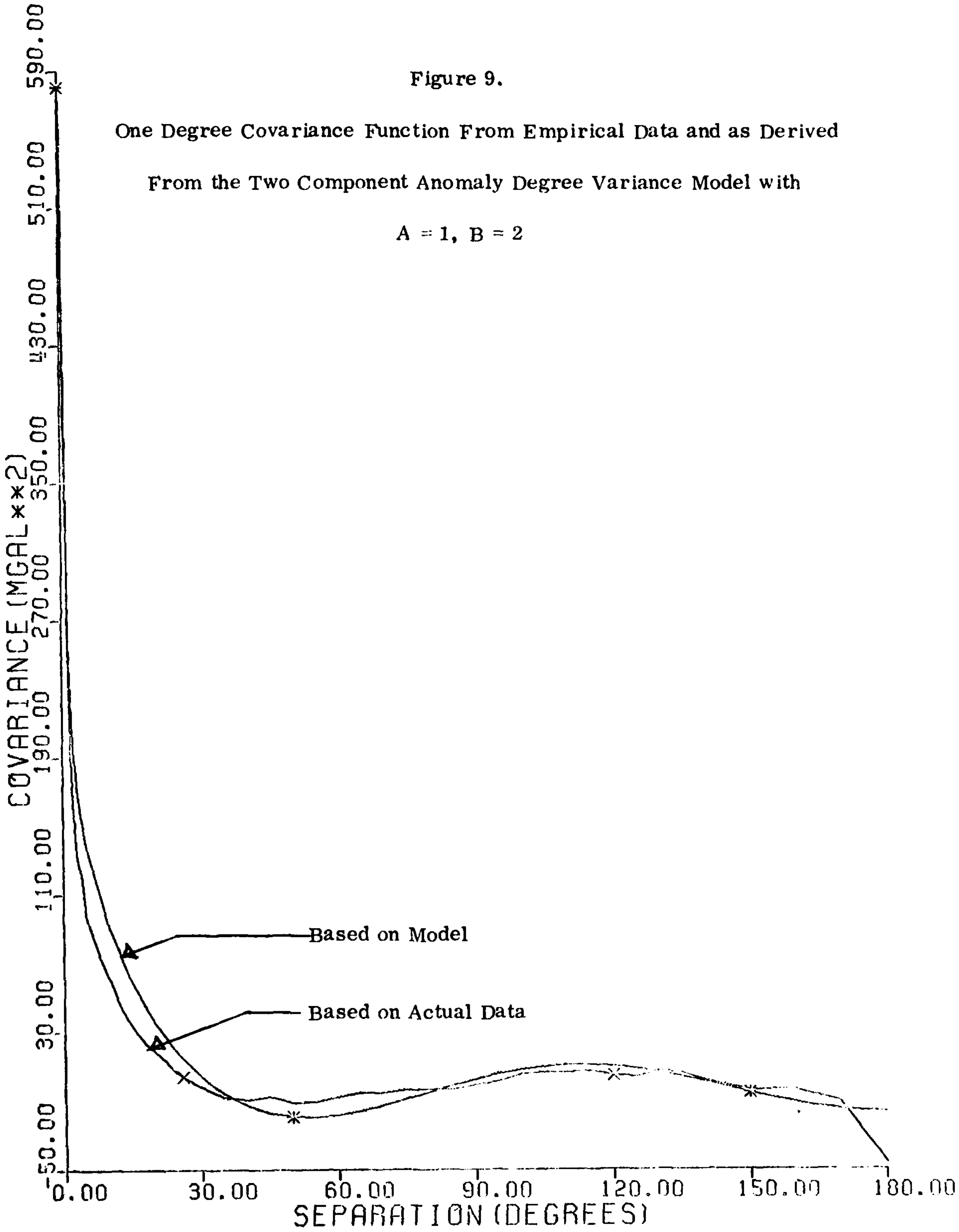


Figure 9.

One Degree Covariance Function From Empirical Data and as Derived From the Two Component Anomaly Degree Variance Model with $A = 1$, $B = 2$

References

Hein, G., and H. Jochemczyk, Reflections on Covariances and Collocation with Special Emphasis on Gravity Gradient Covariances, manuscripta geodaetica, Vol. 4, 45-86, 1979.

Heiskanen, W., and H. Moritz, Physical Geodesy, W. H. Freeman, San Francisco, 1967.

Heller, W., and S. Jordan, Attenuated White Noise Statistical Gravity Model, J. Geophys. Res., 84, 4680-4688, 1979.

Jekeli, C., An Investigation of Two Models for the Degree Variances of Global Covariance Functions, Dept. of Geodetic Science Report No. 275, The Ohio State University, Columbus, 1978.

Kaula, W., Theory of Satellite Geodesy, Blaisdell, Waltham, Massachusetts, 1966.

Kaula, W., Geophysical Inferences from Statistical Analyses of the Gravity Field, in The Changing World of Geodetic Science, Dept. of Geodetic Science Report No. 250, The Ohio State University, Columbus, 1977.

Lerch, F., et. al., Gravity Model Improvement Using Geos-3 Altimetry (GEM 10A and 10B), NASA, Goddard Space Flight Center, Greenbelt, Maryland, 1978.

Moritz, H., On the Computation of a Global Covariance Model, Dept. of Geodetic Science Report No. 255, The Ohio State University, Columbus, 37pp., 1977.

Rapp, R. H., The Prediction of Point and Mean Gravity Anomalies through the Use of a Digital Computer, Dept. of Geodetic Science Report No. 43, The Ohio State University, Columbus, 1964.

Rapp, R. H., Improved Models for Potential Coefficients and Anomaly Degree Variances, J. Geophys. Res., 78, 3497-3500, 1973.

Rapp, R. H., A Global 1° x 1° Anomaly Field Combining Satellite, Geos-3 Altimeter and Terrestrial Anomaly Data, Dept. of Geodetic Science Report No. 278, The Ohio State University, Columbus, 20 pp., 1978.

Tscherning, C., and R. Rapp, Closed Covariance Expressions for Gravity Anomalies, Geoid Undulations, and Deflections of the Vertical Implied by Anomaly Degree Variance Models, Dept. of Geodetic Science Report No. 208, The Ohio State University, Columbus, 90 pp., 1974.

Wagner, C., and O. L. Colombo, Gravitational Spectra from Direct Measurements, NASA Technical Memorandum 79603, 28pp., Goddard Space Flight Center, Greenbelt, Maryland, 1978.